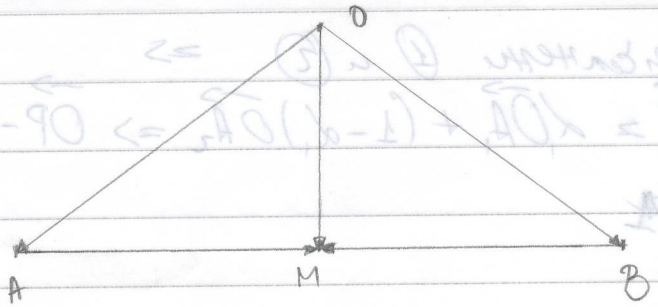
АГ - 15.10.12



$\vec{OM} = \frac{1}{2}(\vec{OA} + \vec{OB})$

$\left.\begin{aligned}\vec{OA} + \vec{AM} &= \vec{OM}\\ \vec{OB} + \vec{BM} &= \vec{OM}\end{aligned}\right\} \Rightarrow 2\vec{OM} = \vec{OA} + \vec{OB} + \underbrace{\vec{BM} + \vec{AM}}_{=0} \Rightarrow$

$A_1 \neq A_2$, P, O

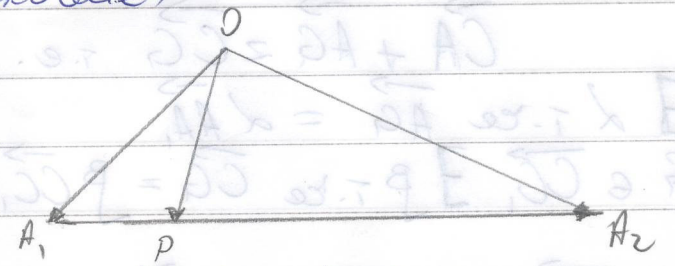
$P \in \text{пр}\, A_1A_2 \iff \exists\, \alpha_1, \alpha_2$ такива че

① $\vec{OP} = \alpha_1\vec{OA_1} + \alpha_2\vec{OA_2}$

② $\alpha_1 + \alpha_2 = 1$

Да се докаже че α_1, α_2 са единствени и не зависят от т. O

Нека $P \in \text{пр}\, A_1A_2$ (от условие)



$\vec{OA_1} + \vec{A_1P} = \vec{OP}$

$\exists\, \lambda : \vec{A_1P} = \lambda\vec{A_1A_2}$

$\vec{OP} = \vec{OA} + \lambda\vec{A_1A_2} = \vec{O_{A_1}} + \lambda(\vec{OA_2} - \vec{OA_1}) = (1-\lambda)\vec{OA_1} + \lambda\vec{OA_2}$

Нека $\alpha_1 = 1 - \lambda$ и $\alpha_2 = \lambda$

Нека са изпълнени ① и ② $\Rightarrow$

$\alpha_2 = 1 - \alpha_1 \Rightarrow \overrightarrow{OP} = \alpha_1 \overrightarrow{OA_1} + (1-\alpha_1)\overrightarrow{OA_2} \Rightarrow \overrightarrow{OP} - \overrightarrow{OA_2} = \alpha_1(\overrightarrow{OA_1} - \overrightarrow{OA_2})$

$\Rightarrow \overrightarrow{A_2P} = \alpha_1 \overrightarrow{A_2A_1}$

Допускаме че $\exists \beta_1$ и β_2 такива че:

③ $\overrightarrow{OP} = \beta_1 \overrightarrow{OA_1} + \beta \overrightarrow{OA_2}$ $\Rightarrow \overrightarrow{A_2P} = \beta_1 \overrightarrow{A_2A_1}$

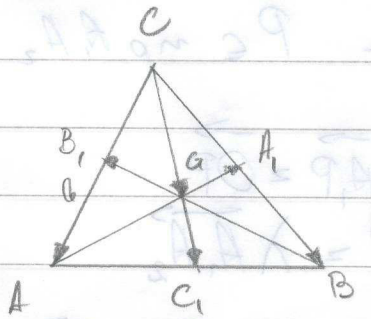
④ $\beta_1 + \beta_2 = 1$

т.е.

$\alpha_1 \overrightarrow{A_2A_1} = \overrightarrow{A_2P} = \beta_1 \overrightarrow{A_2A_1} \Rightarrow (\alpha_1 - \beta_1)\overrightarrow{A_2A_1} = \overrightarrow{0} \Rightarrow$

$\alpha_1 = \beta_1$; $\alpha_2 = \beta_2$ т.е. α единствени

Да се докаже че трите медиани в триъгълника се пресичат в една точка и се делят в отношение 2 към 1 считано от върхът към срещуположната страна. Да се докаже че 3-те медиани са страни от триъгълник



$\overrightarrow{CA} + \overrightarrow{AG} = \overrightarrow{CG}$ т.е.

α т.че $\overrightarrow{AG} = \alpha \overrightarrow{AA_1}$

$\in \overrightarrow{CC_1}$ $\exists \beta$ т.че $\overrightarrow{CG} = \beta \overrightarrow{CC_1}$

$\overrightarrow{CA} + \alpha \overrightarrow{AA_1} = \beta \overrightarrow{CC_1}$ откъдето $\overrightarrow{CA} + \frac{\alpha}{2}(\overrightarrow{AC} + \overrightarrow{AB}) = \frac{\beta}{2}(\overrightarrow{CA} + \overrightarrow{CB}) \Rightarrow$

$\overrightarrow{CA} + \frac{\alpha}{2}(-\overrightarrow{CA} + \overrightarrow{CB} - \overrightarrow{CA}) = \frac{\beta}{2}(\overrightarrow{CA} + \overrightarrow{CB}) \Rightarrow$

$$\left(1-\alpha-\frac{\beta}{2}\right)\vec{CA}+\left(\frac{\alpha}{2}-\frac{\beta}{2}\right)\vec{CB}=\vec{0}$$

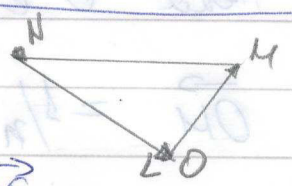
но тъй като $\vec{CA}\neq\vec{CB}\neq\vec{0}\Rightarrow$

$$\left|\begin{array}{l}1-\alpha-\frac{\beta}{2}=0\\ \frac{\alpha}{2}-\frac{\beta}{2}=0\end{array}\right\}\Rightarrow \alpha=\beta=2/3\Rightarrow \vec{CG}=2/3\,\vec{CC_1}$$

Допускаме $BB_1\cap CC_1\in G_1$

по аналогичен начин стигаме до $\Rightarrow$

$$\vec{CG_1}=2/3\,\vec{CC_1}\Rightarrow G_1\equiv G$$

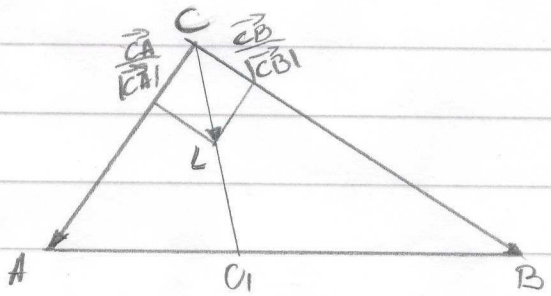


$$\vec{OM}=\vec{AA_1};\ \vec{MN}=\vec{BB_1};\ \vec{NL}=\vec{CC_1}$$

$$\vec{OL}=\vec{OM}+\vec{MN}+\vec{NL}=\vec{AA_1}+\vec{BB_1}+\vec{CC_1}=$$

$$1/2\left(\vec{AB}+\vec{AC}\right)+1/2\left(\vec{BA}+\vec{BC}\right)+1/2\left(\vec{CA}+\vec{CB}\right)\Rightarrow L\equiv O$$

Да се докаже че ъглополовящите в триъгълника се пресичат в една точка и всяка ъглополовяща разделя срещуположната страна в отношение равно на отношението от съседните ѝ страни



$$\vec{CL}=\frac{\vec{CA}}{|\vec{CA}|}+\frac{\vec{CB}}{|\vec{CB}|}$$

$$\vec{CC_1}=\lambda\vec{CL}=\frac{\alpha}{|\vec{CA}|}\vec{CA}+\frac{\alpha}{|\vec{CB}|}\vec{CB}=\frac{\alpha}{|\vec{CA}|}+\frac{\alpha}{|\vec{CB}|}=1\Rightarrow$$

$$\alpha = \frac{|\vec{CA}||\vec{CB}|}{|\vec{CA}|+|\vec{CB}|}$$

$$\vec{CC_1} = \frac{|\vec{CB}|}{|\vec{CA}|+|\vec{CB}|}\vec{CA} + \frac{|\vec{CA}|}{|\vec{CA}|+|\vec{CB}|}\vec{CB}$$

Точки $A_1 \dots A_n$, A_n - центроид

$$\vec{MA_1} + \dots + \vec{MA_n} = \vec{0}$$

Да се докаже че има точно 1 центроид

Нека O е произволна точка в равнината

$$\vec{OM} = \frac{1}{n}(\vec{OA_1} + \dots + \vec{OA_n})$$

$$\vec{MA_1} + \dots + \vec{MA_n} = (\vec{OA_1} - \vec{OM}) + \dots + (\vec{OA_n} - \vec{OM}) = (\vec{OA_1} + \dots + \vec{OA_n}) - n\vec{OM} =$$

$$= \vec{0}$$

